About the Relativistic Invariant Wave Function of a particle

Yurasov Nickolay Ilyich[1] yurasovni@bmstu.ru

SPIN-code: 2490-9108

[1]*BMSTU, Moscow, Russia*

Abstract. The wave function has investigated as function of action. We have considered different forms of relativistic action for particle such as with electromagnetic field, with gravitation, with spin polarization. We have represented original form of equation of Schrödinger and boundary conditions.

Keywords: wave function, action, charge particle, gravitation charge, scalar potential, vector potential, boundary conditions

Introduction. Using the sentence of Feynman [1] about the physical meaning of the wave function phase as $\frac{S}{\hbar}$, where S is an action, $\hbar$ is the Dirac's constant we have considered original form of the Schrödinger's equation.

Additive property of action allows us include some interactions such as electromagnetic and gravitation. The action may consist of action of the free particle, of action of charge particle and of action of massive particle and some other actions. Using the sentence of Feynman [1], operator of the Schrödinger, we get original form or matrix form. This representation of elements of wave function of a particle is new uninvestigated problem and it is aim of our analysis.

Methods and materials; results. Consider equation of Schrödinger for wave function ψ in form [2]

$$i\hbar\psi_{,t} - \hat{\mathrm{H}}\psi = 0\ , \tag{1}$$

where down index $(,t)$ after ψ has meaning differentiability on t, $\hat{\mathrm{H}}$ is Hamiltonian. Represent wave function in form agree with the sentence of Feynman [1]

$$\psi = \exp\left(\frac{S}{\hbar}\right) \tag{2}$$

If we have considered model without spin and gravitation, we have formula

$$S = -mc^2\int_{t_1}^{t_2}(1-\beta^2)^{\frac{1}{2}}dt - \frac{q}{c}\int_{t_1}^{t_2}\varphi dt + q\int_{t_1}^{t_2}\beta_u A_u dt\ , \quad u = (x, y, z), \tag{3}$$

where m, c – are mass, velocity of light; t is moment of time, $v_u = \beta_u c$ is velocity of particle in the laboratory system, q is charge, φ is scalar potential, A_u is vector potential. If we consider model where it's gravitation charge $q_g$yet, we must supply two terms in formula (3)

$$\Delta S = \frac{q_g}{c}\int_{t_1}^{t_2}\varphi_g dt - q_g\int_{t_1}^{t_2}\beta_u A_{gu} dt, \tag{4}$$

where φ_g is scalar potential, A_{gu} is vector potential of the gravitation field [3].

In model with spin unequal zero, the wave function (2) has matrix form

$$\psi = \begin{pmatrix}\psi_1\\ \vdots\\ \psi_k\end{pmatrix}, k=\begin{cases}2l+1\\ 2\left(l+\frac{1}{2}\right)\end{cases}, l = 0,1,2,\ldots \tag{5}$$

Therefore, we have action in matrix form also

$$S = \begin{pmatrix}S_1\\ \vdots\\ S_k\end{pmatrix}, k=\begin{cases}2l+1\\ 2\left(l+\frac{1}{2}\right)\end{cases}, l = 0,1,2,\ldots \tag{6}$$

and the Schrödinger's equation has new form

$$iS_{,t} - \widehat{\mathrm{H}}S = 0 \tag{7}$$

Now we must determine boundary conditions. They has follow forms

$$S_{1_F} = S_{2_F} \quad \cup \quad \left(\frac{\partial S_1}{\partial n}\right)_F = \left(\frac{\partial S_2}{\partial n}\right)_F \tag{8}$$

where F is boundary surface and n is normal to surface. The boundary conditions (8) agree with standard boundary conditions

$$\psi_{1_F} = \psi_{2_F} \quad \cup \quad \left(\frac{\partial \psi_1}{\partial n}\right)_F = \left(\frac{\partial \psi_2}{\partial n}\right)_F \quad . \tag{9}$$

if $\widehat{\mathrm{H}}$ is the differential operator.

Conclusion. The influence of transition from the wave function to action has investigated. The complex relativistic actions have considered. The quantum equation and the boundary conditions have found. Taking account additive property of action we have demonstrated possibility new analyze of Schrödinger's equation as equation for action.

References

[1] Feynman R.P., Leighton R.B., Sands M. The Feynman lectures on physics. Vol.2. Addison Wesley Publishing Company, Inc. Reading, Massachusetts, Palo Alto, London, 1964

[2] Dirac P.A.M., The fundamental equations of quantum mechanics, *Proceedings of the Royal Society A*, 1925, vol. 109, pp. 642-653

[3] Brillouin L., Relativity reexamined, Academic Press, N.Y. and London, 1970