Quasi-gravitation rather than pseudo-telepathy: Geometric model of quantum navigation during (anti)search on a plane

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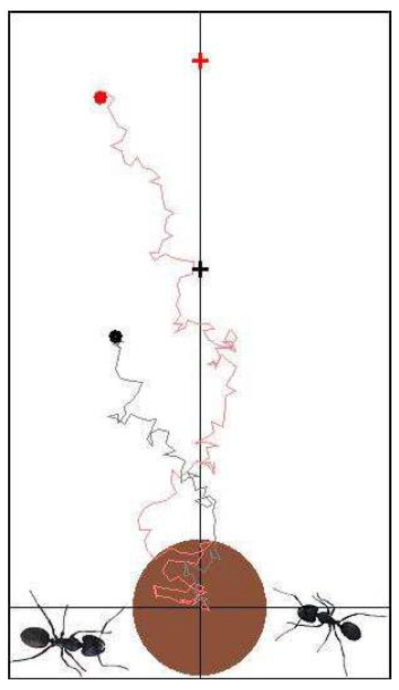
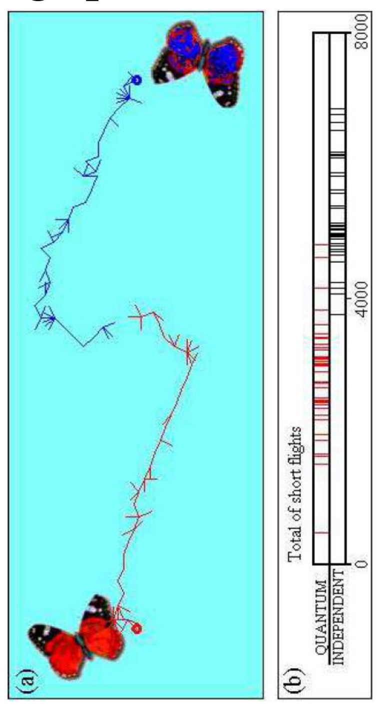
Introduction

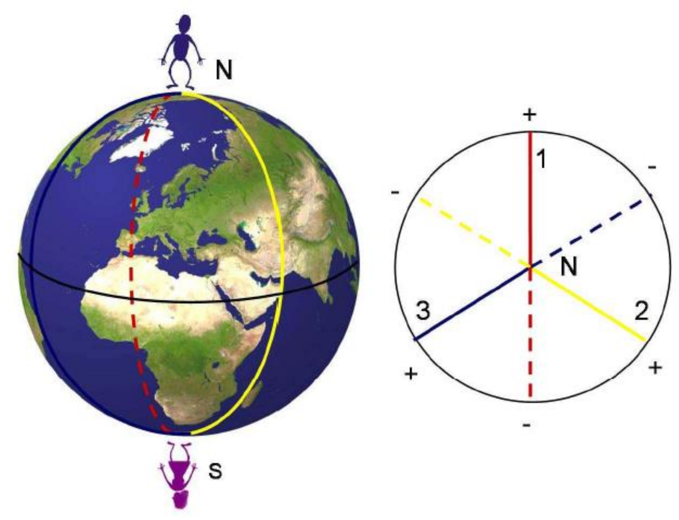
Quantum games – area in quantum physics that studies variants of games known from classical game theory, enhanced by resources of non-local correlations (entanglement) distributed among the participants. It serves for deeper understanding of the nature of quantum correlations and their possible applications in quantum communication and quantum cryptography tasks.

Quantum pseudo-telepathy – decrease (or elimination) of need of communication between participants in order to achieve the goal of the (cooperative) quantum game [1].

Notable games involving spatial movement that demonstrate quantum advantage:





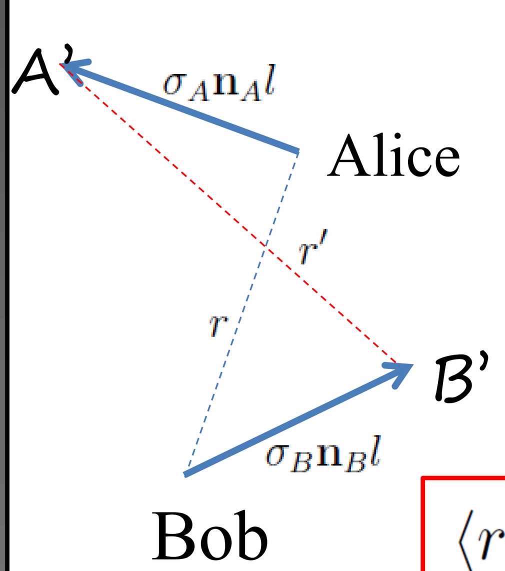


joint search on a sphere (quantum advantage ~ 1.07) [3]

optimally pushing a pebble + joint random search with learning (quantum advantage ~ $\sqrt{2}$) [2]

Goal of present work: provide a meaningful geometric explanation of advantage given by use of quantum resources in quantum games involving spatial movement.

Joint random walks on an infinite plane

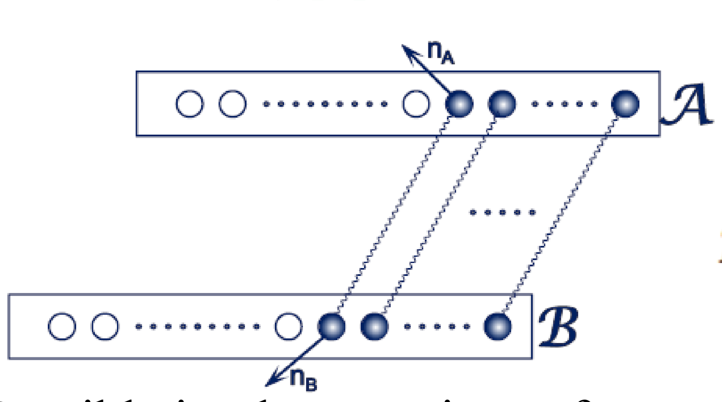


Alice and Bob move in a sequence of steps of length l in random directions $\mathbf{n}_{A,B}$ upgraded by random signs $\sigma_{A,B} = \pm 1$. Participants cannot communicate with each other and have no common landmarks (real-world motivation – submarines in ocean), but have a distributed resource of pairs of entangled particles. The goal is either to find each other or to get as far from each other as possible.

$\boxed{\langle r'^2 \rangle_0^{(1)} = r^2 + 2l^2}$ - after step 1

1. Classical random walk (**"0" protocol**) – the signs and directions of each step are distributed uniformly.
2. Quantum random walk – the directions are chosen randomly, but the signs are chosen according to the results of quantum measurements performed on entangled spin-1/2 particles in singlet states distributed between Alice and Bob (double spin projections on the directions of displacements are measured) [4].

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\Big(|\mathbf{n}\rangle_a \otimes |-\mathbf{n}\rangle_b - |-\mathbf{n}\rangle_a \otimes |\mathbf{n}\rangle_b\Big)$$



$$p(\sigma_A, \sigma_B | \mathbf{n}_A, \mathbf{n}_B) \doteq \frac{1}{4}\Big[1 - \sigma_A \sigma_B (\mathbf{n}_A \cdot \mathbf{n}_B)\Big]$$

Possible implementations of quantum random walks:

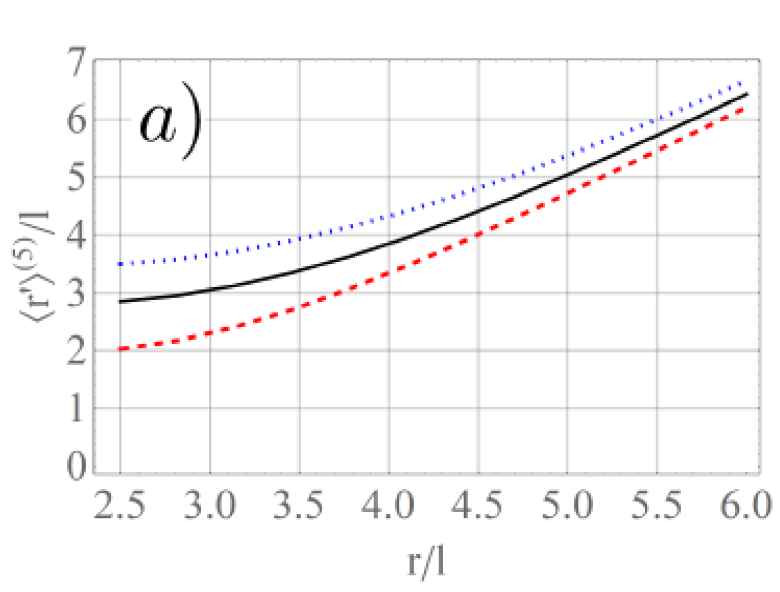
1) **"+" protocol**: Alice moves along $\sigma_A \mathbf{n}_A l$, while Bob moves along $-\sigma_B \mathbf{n}_B l$

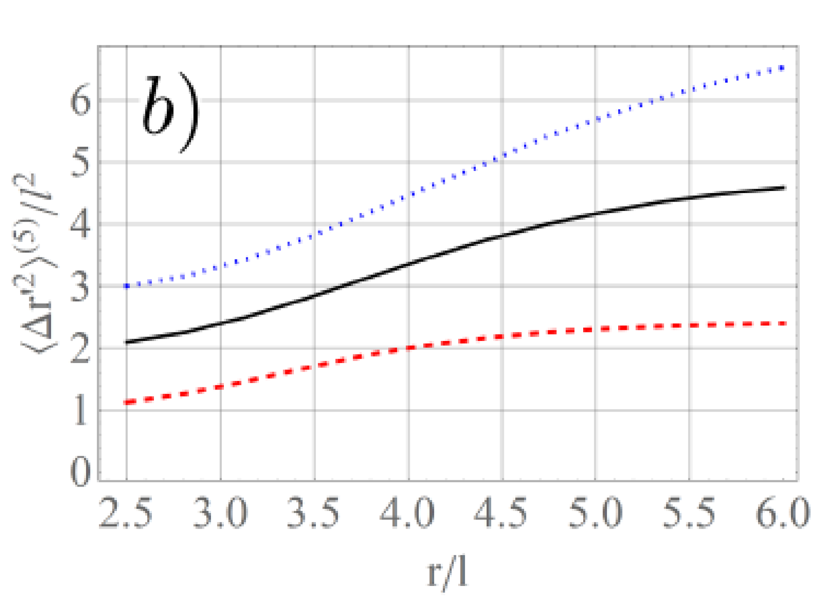
$\boxed{\langle r'^2 \rangle_+^{(1)} = r^2 + l^2}$ (effective attraction w/r to classical case)

2) **"-" protocol**: Alice moves along $\sigma_A \mathbf{n}_A l$, while Bob moves along $\sigma_B \mathbf{n}_B l$

$\boxed{\langle r'^2 \rangle_-^{(1)} = r^2 + 3l^2}$ (effective repulsion w/r to classical case)

Analysis of many-step sequence confirms reduced fluctuations for "+" protocol and increased fluctuations for "-" protocol:





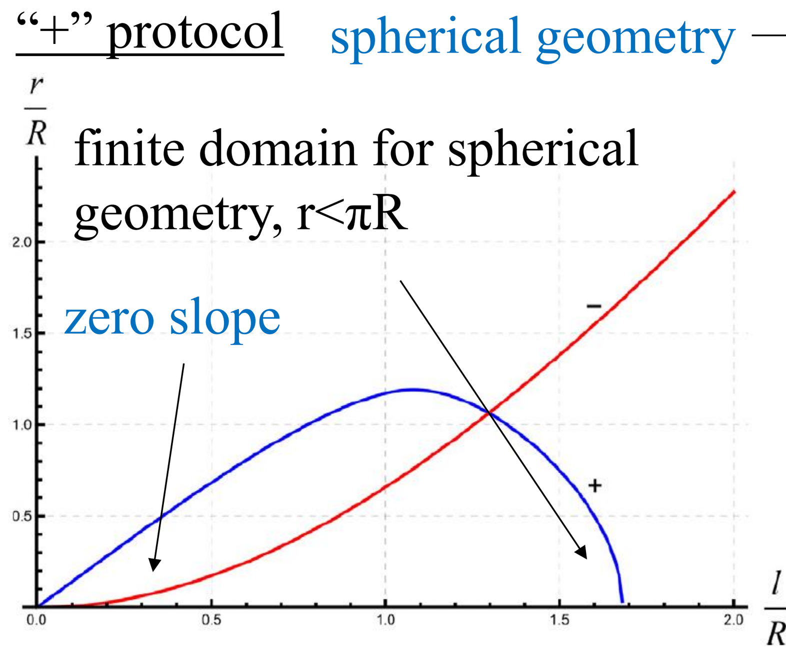
Effective curvature induced by quantum random walks

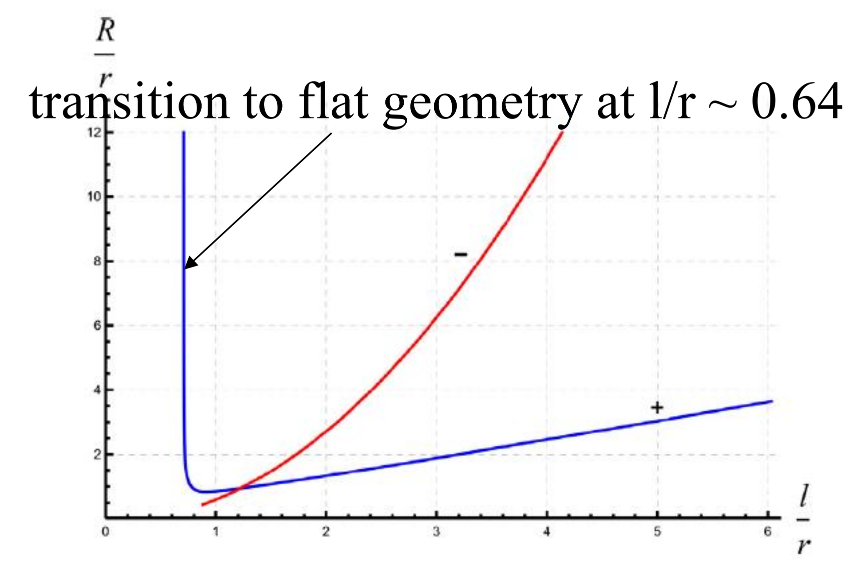
Quantum random walks on a plane are equivalent to classical random walk on a surface with curvature radius R, which is defined separately in each step:

"geometry" "physics" $R \mapsto iR$ "geometry" "physics"

$\boxed{R^2\langle \arccos^2[\cos(r'/R)]\rangle = r^2 + l^2} \longrightarrow \boxed{R^2\langle \mathrm{arccosh}^2[\cosh(r'/R)]\rangle = r^2 + 3l^2}$

"+" protocol spherical geometry $\longrightarrow$ hyperbolic geometry "-" protocol



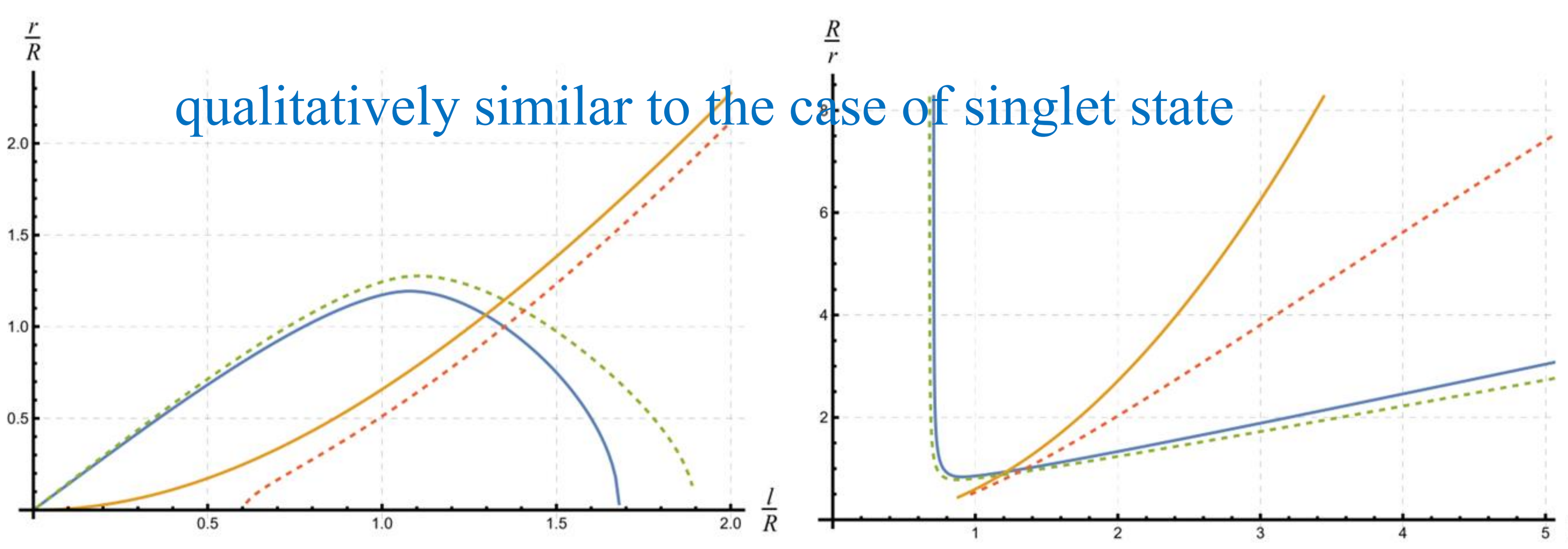


Hyperbolic geometry is valid for all values of l/r.

By pre- and post-selecting singlet states, it is possible to enhance the effect of correlations:

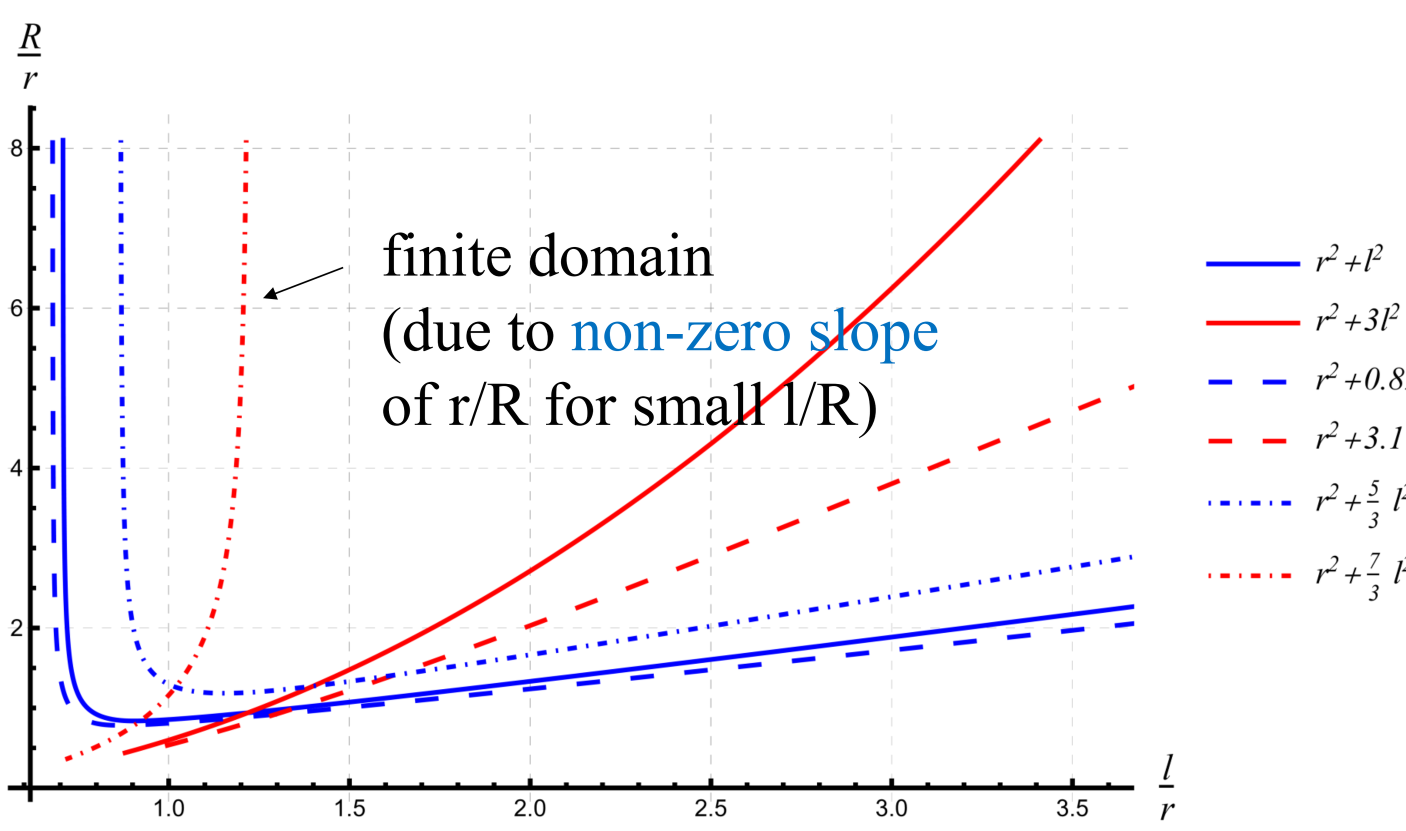
$$p_{PPS}(\sigma_A, \sigma_B | \mathbf{n}_A, \mathbf{n}_B) = \frac{1}{4}\left[1 - \sigma_A \sigma_B \frac{2(\mathbf{n}_A \cdot \mathbf{n}_B)}{1 + (\mathbf{n}_A \cdot \mathbf{n}_B)^2}\right]$$

$R^2\langle \arccos^2[\cos(r'/R)]\rangle = r^2 + 2(\sqrt{2}-1)l^2$ $\qquad$ $R^2\langle \mathrm{arccosh}^2[\cosh(r'/R)]\rangle = r^2 + 2(3-\sqrt{2})l^2$



Other types of correlations also allow interpretation in terms of effective curvature. Maximal classical correlations are achieved in Werner state for p=1/3: $\hat{\varrho}(p) = \frac{1-p}{4}\hat{1} \otimes \hat{1} + p|\Psi\rangle\langle\Psi|$

$R^2\langle \arccos^2[\cos(r'/R)]\rangle = r^2 + \frac{5}{3}l^2$ $\qquad$ $R^2\langle \mathrm{arccosh}^2[\cosh(r'/R)]\rangle = r^2 + \frac{7}{3}l^2$



For non-maximally entangled states, there is a maximum value of l/r beyond which the hyperbolic model is inapplicable.

Conclusion and outlook

Correlations observed in quantum random walks on plane surface can be explained by introduction of effective spherical or hyperbolic geometry. However, for latter case the model is universally applicable only to maximally entangled states, otherwise it has limited domain of step length and initial placement of participants.

Results of preliminary investigations of geometric models for many-body correlations emerging in random walk of three participants were reported in [5].

References

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2. Summhammer, arXiv:quant-ph/0503136v2 (2006).
3. Brukner et al., arXiv:quant-ph/0509123v1 (2005).
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5. Rostom, Tomilin, Il'ichov, JETP Letters **120**, 728 (2024).