Vacuum Entanglement & Horizon

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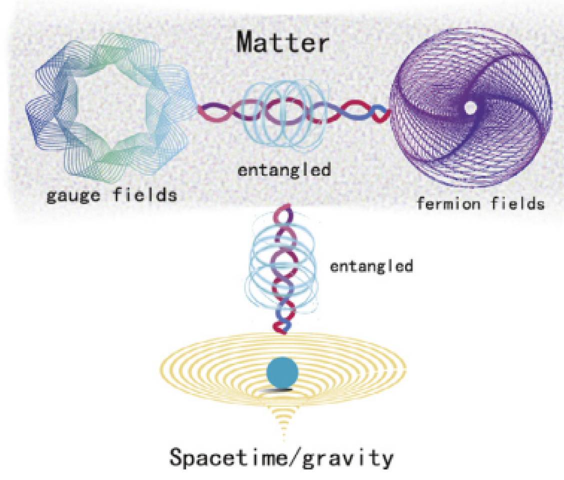
Aim: The short-distance behavior of a QFT is determined by its vacuum. We study the effects of gravity, Hawking radiation, and local Unruh radiation on the vacuum entanglement of a massless Fermionic field outside a Schwarzschild black hole using two pointlike detectors along various trajectories.

" What is it ?" & " Why bother with it ?"

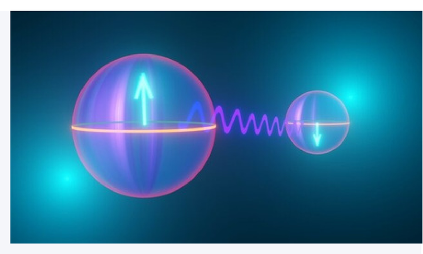
- Entanglement: L and M are entangled if you can not write the full state, |ψ>, in a separable (|ψ> = Σ M |L>|R> ; Rank[M]=1) form .
- Matter fields in different regions of spacetime, including causally disconnected regions, are entangled.
- Different parts of spacetime itself are entangled, and it has been argued that this entanglement acts as a kind of glue for spacetime.
- Not only is the presence of entanglement necessary, but a sufficient amount is also needed for the smoothness of horizons.
- The notion of entanglement, which depends on Hawking radiation, also depends on how you factorize the Hilbert space, if possible.
- Apart from black hole physics, it is also crucial for various other branches, such as quantum teleportation, photosynthesis,...etc.

|Universe⟩ =



Zeng-Bing Chen (2023)

Setup



- We take 2 point like two-level systems [UDWs] which are initially non-entangled.
- Put them in causally disconnected regions and see after some time they got entangled.
- Since there is no communication, it means they have harvested entanglement present in field.

Interaction Hamiltonian

$$\hat{H}_j^{\rm int}(\tau_j) = \lambda_j \chi_j(\tau_j)\hat{\mu}(\tau_j) : \bar{\psi}(x(\tau_j))\psi(x(\tau_j)) :$$

Entanglement Measure:

Mutual Information(total correlations)

$$I[\rho_{AB}] := S(\rho_{AB}|\rho_A \otimes \rho_B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$$

Concurrence(Entanglement)

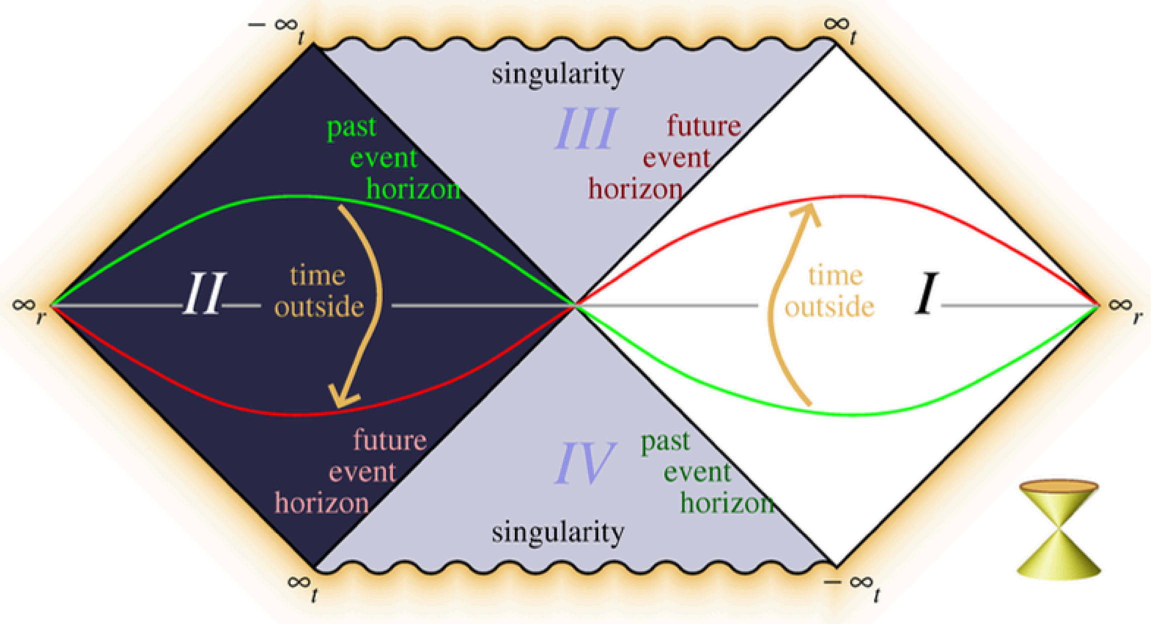
$$\mathcal{C}[\rho_{AB}] = 2\max[0, |\mathcal{M}| - \sqrt{\mathcal{L}_{AA}\mathcal{L}_{BB}}] + \mathcal{O}(\lambda^4)$$

M: Nonlocal terms; L: diagonal terms in density matrix AB

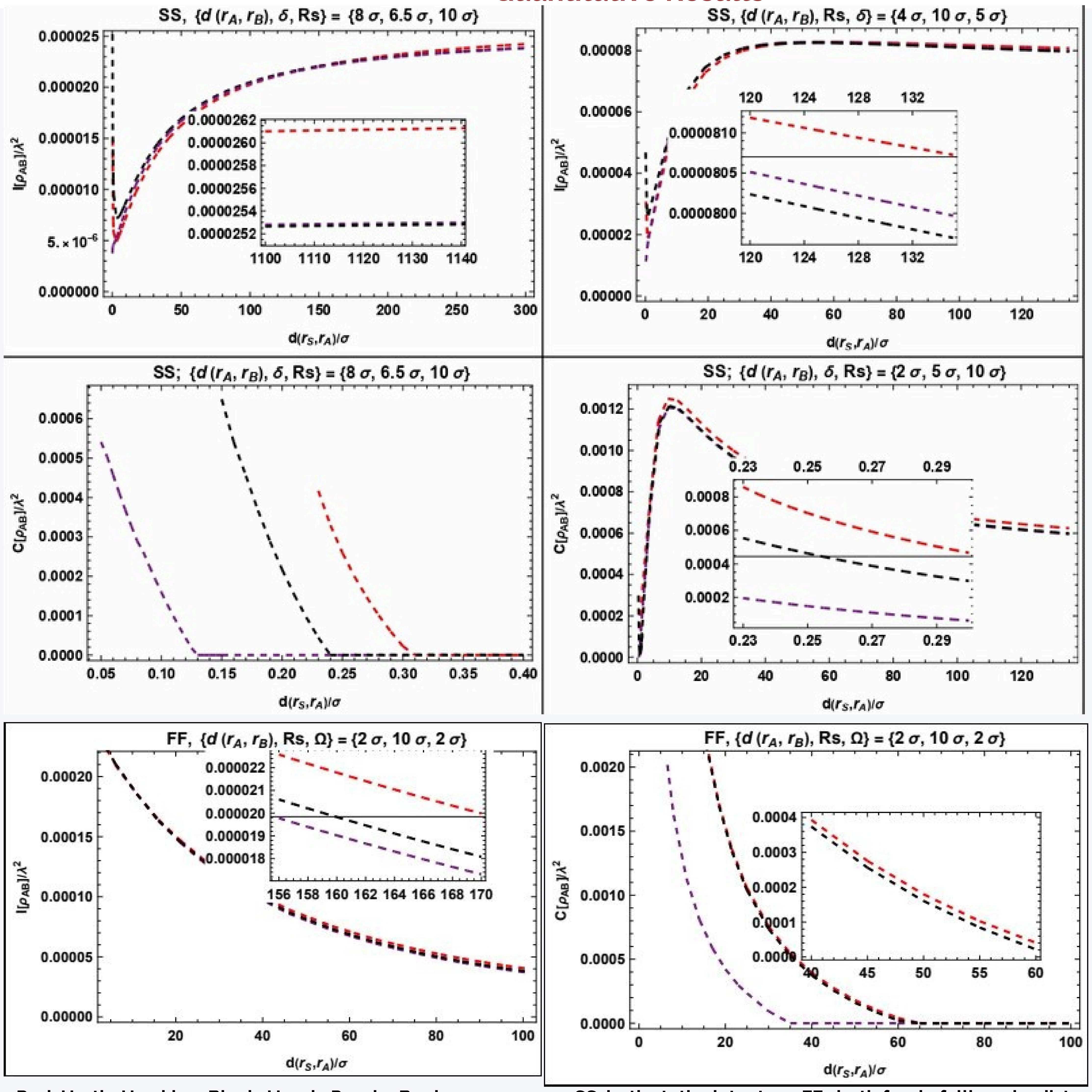
Earlier works

- The concept of entanglement began with Schrödinger in 1932; however, the entanglement harvesting protocol had to wait until 1990s (Valentini et. al.1991)
- In 2002, the relationship between acceleration and entanglement emerged with the combination of quantum entropy, vacuum entanglement, and quantum teleportation, marking the dawn of relativistic quantum information.
- Afterward, numerous prospects such as entanglement of matter fields in cosmological spacetimes, near black hole entanglement, and the effect of thermal baths were developed by various people.
- Importantly, people have observed the death of entanglement near the horizon for static detectors coupled with a real massless scalar field.

Adopted from Science J. for kids



Quantitative Results



Red: Hartle Hawking, Black: Unruh, Purple: Boulware vacuum; SS: both static detectors; FF : both freely falling; da: distance from horizon; σ: standard width of Gaussian switching; dab: distance between detectors; Ω: Energy gap; Rs: Schwarzschild radius; δ: delay

Transition Rate(Static detector)

$$\dot{\mathcal{F}} = \int_{-\infty}^{\infty} d\Delta\tau e^{-i\Omega\Delta\tau} W_{\psi}^{\alpha}(x(\tau), x(\tau'))$$

$$\dot{\mathcal{F}}_{HHI} = \frac{\Omega}{\pi(e^{4\pi\kappa r_S \Omega} - 1)}$$

Thermal with twice magnitude than real massless scalar field

$$\dot{\mathcal{F}}_{Unruh} = \frac{1}{2}\left[2 \times \frac{1}{4\pi^2 r_S \kappa} \log \frac{e^{4\pi\kappa r_S \Omega}}{(e^{4\pi\kappa r_S \Omega} \pm 1)}\right]$$

Helmholtz free energy of bosons/fermions

(A departure from thermality)

- Conclusion : The transition rate in the HHI is thermal with bosonic statistics, while in the Unruh vacuum, the detector detects the Helmholtz free energy of the Fermionic/bosonic field. There is no entanglement death for sufficiently separated static detectors coupled with a massless Fermionic field, which is in contrast with the scalar field. There is no entanglement death for freely falling detectors with any separation at the time of switching peak, suggesting that the death is caused by gravitational redshift. Plots also suggest that the anti Hawking effect lead to the enhancement of entanglement, while the Hawking radiation diminish it.

Bibliography

1. B. Reznik, Entanglement from the vacuum, Found. Phys. 33 (2003) 167-176..
2. Mann et al. 2012 ; Classical and Quantum Gravity 29(22):220301
3. Nitesh and Sanved 2025 (arXiv:2504.06887 [gr-qc])