Vacuum pressure and the flow of time beneath the Earth's surface

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Introduction

- The non-zero vacuum pressure is an element of cosmological models, resulting from the solution of Einstein's equations. We consider the possibility of determining the vacuum pressure from the properties of a local gravitational system. Sakharov [1] suggested that gravity arises from quantum field theory in much the same sense that hydrodynamics or elasticity theory arises from molecular physics. He believed that the curvature of space leads to the 'metric elasticity' of space, i.e. to generalized forces that counteract its curvature. The action term of Einstein's geometrodynamics is identified with the change in the action of quantum fluctuations of the vacuum [2]-[4].
- From general relativity follows that gravitational mass of bodies placed in confined volume, is less than the sum of the gravitational masses of these bodies, dispersed over infinite distance. The matter, located more compactly, distorts the space in the local domain in a greater degree, however, creating smaller gravitational mass in comparison with the same amount of matter, distributed over a greater volume [5, 6]. In [7] this is interpreted as the appearance of negative binding energy in the gravitational system. In present work this phenomenon is explained by transfer of energy into the gravitational field, which results in the deformation of space. Accumulation of energy during deformation demonstrates its elasticity. These properties of gravity are taken into account when determining the vacuum pressure [8, 9, 10].
- The time dilation in the outer region near the Earth's surface, corresponding to the Schwarzschild metric, has been experimentally confirmed [11]. The resulting vacuum pressure corresponds to the solution of Einstein's equations for a spherical gravity source [8, 12]. In the present work, using this solution, the flow of time at the center of the Earth is determined.

Solution of Einstein equations for spherical source

The general static, spherically symmetric line element in spherical coordinates $x^i = (ct, r, \theta, \varphi)$ is

$$ds^2 = c^2 f(r) dt^2 - \frac{dr^2}{h(r)} - r^2(d\theta^2 + \sin^2\theta \, d\phi^2). \qquad (1)$$

with metric functions $f(r)$ and $h(r)$. The stress-energy tensor of a static, spherically symmetric distribution of matter with density ρ and isotropic pressure p is described by the diagonal matrix

$$T_i^i = diag(c^2\rho, -p, -p, -p). \qquad (2)$$

Metric functions are sought from the Einstein equations reduced [13] to

$$G^t{}_t = \frac{1}{r^2}\frac{d}{dr}[r(1-h)] = \chi T_1^1, \qquad (3)$$

$$G^r{}_r = -\frac{h}{rf}\frac{df}{dr} + \frac{1}{r^2}(1-h) = \chi T_2^2 \qquad (4)$$

and the covariant conservation equation

$$\nabla_i G^i_k = \frac{dp}{dr} + \frac{p+\rho}{2f}\frac{df}{dr} = 0 \qquad (5)$$

with constant

$$\chi = \frac{8\pi\gamma}{c^4}, \qquad (6)$$

where γ is gravitational constant.

Solution of Einstein equations for spherical source

Integration of equation (3) yields

$$h(r) = 1 - \frac{\chi}{r}\int_0^r T_1^1 y^2 dy, \tag{7}$$

where y is a variable.

The logarithm of the function $f(r)$ [8,9] has the form

$$\ln[f(r)] = \ln\left(1 - \frac{\chi}{r}\int_0^r T_1^1 y^2 dy\right) - \chi \int_r^a \frac{(T_1^1 - T_2^2)z}{1 - \frac{\chi}{z}\int_0^z T_1^1 y^2 dy} dz, \tag{8}$$

where z is a variable. If the non-zero values region of the energy-momentum tensor is limited by a radius a, then for $r \geq a$ we have metrical functions

$$f(r) = h(r) = 1 - \frac{\chi}{r}\int_0^a T_1^1 y^2 dy, \tag{9}$$

which correspond to the Schwarzschild metric for a spherical body with mass

$$M = \frac{4\pi}{c^2}\int_0^a T_1^1 r^2 dr\,. \tag{10}$$

Pressure in low gravitating sphere

Integration is performed in (10) for the element of volume $dV_c = 4\pi r^2 dr$, which corresponds to the coordinate frame, whereas in proper frame the given element of space volume [14] is determined as

$$V = \sqrt{\det\left[\frac{g_{1p}g_{1q}}{g_{11}} - g_{pq}\right]}\, dx^2 dx^3 dx^4 \qquad (11)$$

with $p, q = 2{,}3{,}4$ and for metric (1) it is

$$dV_p = 4\pi r^2 h^{-1/2} dr. \qquad (12)$$

Inequality $h < 1$, means that the gravitational mass of body is less than the sum of individual gravitational masses its constituent elements. This difference is also called a gravitational mass defect.

The gravitational impact on the vacuum is determined as the relation of difference between proper energies of two spherical bodies with identical gravitational mass and constant densities to the change of proper volume of space [9]. With the densities ρ_1, ρ_2, and radii a_1, a_2 $(a_1 < a_2)$ this mass is

$$M = \frac{4}{3}\pi\rho_1 a_1^3 = \frac{4}{3}\pi\rho_2 a_2^3. \qquad (13)$$

Pressure in low gravitating sphere

The volume of spherical body in proper frame is obtained by integration of elements dV_p with (2), (7) and amounts to

$$\mathrm{V}_{\mathrm{int}}^{p}(a) = \int_0^a \frac{4\pi r^2}{\left(1 - \frac{1}{3}\mathrm{c}^2\chi\rho r^2\right)^{1/2}} dr. \tag{14}$$

For small space curvature inside the sphere, i.e. with $c^2\chi\rho a^2 << 1$, representation of the expression under the integral into a formal power series turns out to be

$$\mathrm{V}_{\mathrm{int}}^{p}(a) = \frac{4\pi}{3}a^3 + \frac{2}{15}\pi c^2\chi a^5\rho + \ldots\ . \tag{15}$$

The mass of body in this frame or the proper mass is

$$M^p = \mathrm{V}_{\mathrm{int}}^{p}(a)\rho\,. \tag{16}$$

The difference of proper masses of two bodies (13) is written as follows:

$$\Delta M^p = M_1^p - M_2^p = \frac{1}{20}\pi c^2\chi a_1^3\rho_1 M\left(\frac{1}{a_1} - \frac{1}{a_2}\right). \tag{17}$$

Pressure in low gravitating sphere

Due to equality of gravitational masses of both bodies, the space distortion in the area $r > a_2$, created by them, will be identical. Let's find the difference between the volumes in the proper frame, which are set in coordinate frame by the condition $r < a_2$. This volume for the first body is the sum of this body's own volume and the peripheral area $a_1 < r < a_2$, namely,

$$\mathrm{V}_1^p = \mathrm{V}_{\mathrm{int}}^p(a_1) + \mathrm{V}_{ext}^p(a_1, a_2), \tag{18}$$

where the second term is given by

$$\mathrm{V}_{ext}^p(a_1, a_2) = \int_{a_1}^{a_2} 4\pi r^2 h^{-1/2}\, dr. \tag{19}$$

with $h(r)$ defined by (9) on this interval. This expression is rewritten as

$$\mathrm{V}_{ext}^p(a_1, a_2) = \int_{a_1}^{a_2} \frac{4\pi r^2}{\sqrt{1 - \frac{c^2\chi\rho a^3}{3r}}}\, dr. \tag{20}$$

Breaking the expression under integral into the formal power series, in case of $c^2\chi\rho a^3/r << 1$, получим

$$\mathrm{V}_{ext}^p(a_1, a_2) = \frac{4\pi}{3}(a_2^3 - a_1^3) + \frac{1}{4}c^2\chi M(a_2^2 - a_1^2)\,. \tag{21}$$

Pressure in low gravitating sphere

As a result, the volume (18) taking into account (14) will amount to

$$\mathrm{V}_1^p = \frac{4\pi}{3} a_2^3 + \frac{1}{20} c^2 \chi M \left(5a_2^2 - 3a_1^2\right). \tag{22}$$

In area $r < a_2$ there is the second body, whose proper volume for the weak gravitational field according to (15) is

$$\mathrm{V}_2^p = \mathrm{V}_{\mathrm{int}}^p(a_2) = \frac{4\pi}{3} a_2^3 + \frac{1}{10} c^2 \chi a_2^2 M . \tag{23}$$

The difference between the proper volumes, confined within the radius a_2, coordinate frame, will be

$$\Delta\mathrm{V}^p = \mathrm{V}_1^p - \mathrm{V}_2^p = \frac{3}{20} c^2 \chi M (a_2^2 - a_1^2). \tag{24}$$

The ratio of change in the proper energy of the spherical body $\Delta\mathrm{E}^p = c^2 \Delta M^p$ to the change of its volume for small $\Delta a = a_2 - a_1$ its gravitational mass taking (17) into consideration yields

$$\wp = \frac{\Delta\mathrm{E}^p}{\Delta\mathrm{V}^p} = \frac{1}{3} c^2 \rho . \tag{25}$$

Pressure in low gravitating sphere

The relationship between density and pressure in expression (25) coincides with the state equation of ideal relativistic gas [5] and photon gas [15]. With increasing masses defect, the difference between the proper volume of the sphere and its volume in a remote frame

$$\Delta V = (V_1^p - V^c) - (V_2^p - V^c) = \Delta V^p \tag{26}$$

increases. In the theory of elasticity $\wp$ corresponds to the pressure of an perfect liquid. Positive pressure of gravity field characterizes the gravitational impact of matter on the vacuum, which lies in its constraint. In accordance with Sakharov's idea of space "metric elasticity" the vacuum pressure compensating it has the opposite direction and in the static case will be

$$p_v = -\wp \,. \tag{27}$$

It can be considered as the average vacuum pressure inside the sphere, provided that it is the total density of matter and non-gravitational fields. It is assumed that the source of the gravitational field is precisely the vacuum pressure, which is included in the energy-momentum tensor.

Solution for pressure $p = -(1/3)c^2\rho$

At constant density and pressure corresponding to (27),

$$p_v = -\frac{1}{3}c^2\rho \tag{28}$$

equations (3)-(5) have solution [8, 12]:

$$h(r) = 1 - \tfrac{1}{3}c^2\chi\rho r^2, \tag{29}$$

$$f(r) = const. \tag{30}$$

The metric functions f and h must satisfy the external Schwarzschild solution (9) in vacuum, so the boundary conditions will be as follows

$$f(a) = h(a) = 1 - \frac{r_0}{a} \tag{31}$$

with gravitational radius r_0. Since the function f is constant inside the sphere $r \le a$, it has the same meaning in this area

$$f(r) = 1 - \frac{r_0}{a}. \tag{32}$$

For the inner region, the line element (1) with metrical functions (29), (32) takes the form

$$ds^2 = c^2\left(1 - \frac{r_0}{a}\right) dt^2 - \frac{dr^2}{1 - \frac{1}{3}c^2\chi\rho r^2} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{33}$$

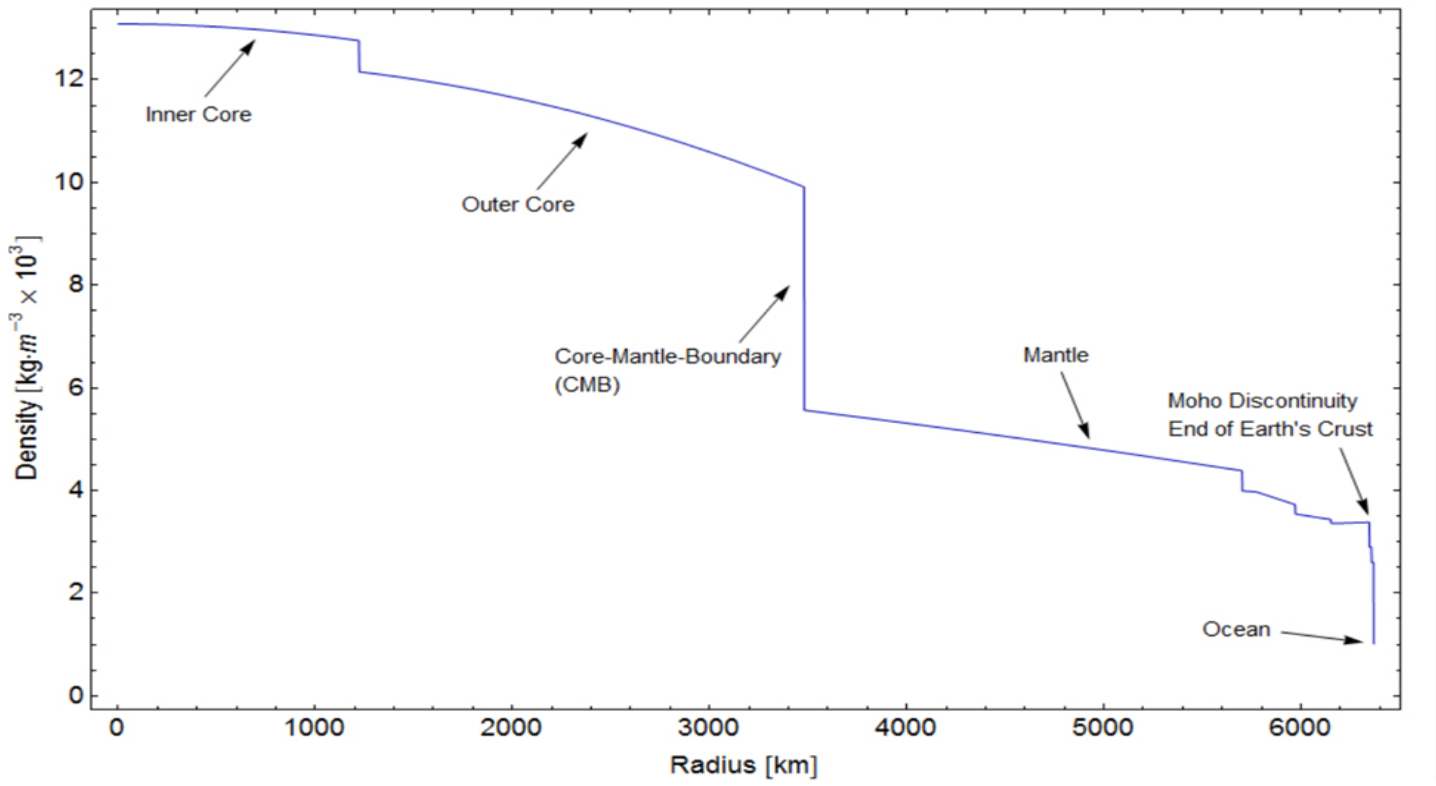
The resulting metric corresponds to the Friedmann space-time with constant scale factor at constant space curvature $k = (1/3)\chi c^2\rho$ and time

$$\tau = \left(1 - \tfrac{1}{3}\chi c^2\rho a^2\right)^{1/2} t. \tag{34}$$

The time dilation will be the same inside the sphere and at the surface.

The course of time below the Earth’s surface

- The Earth consists of several regions, the density in which differs significantly, Fig. 1.



- Figure 1: A graph of the dependence of the Earth’s density on the distance to its center was built using data from [16].

The course of time below the Earth's surface

Let us first estimate the flow of time at the center of the Earth. We consider the core density to be approximately $\rho_{co} = 11000\ \mathrm{k}g/m^3$ and the density of the outer region, which includes the mantle and crust, $\rho_{out} = 4400\ \mathrm{k}g/m^3$. In this case, the gravitational field inside the Earth will be the sum of the fields of two spheres with $\rho_1 = \rho_{out}$, $a_1 = a_E = 6378\ \mathrm{k}m$ and $\rho_2 = \rho_{co} - \rho_{out} = 6600\ \mathrm{kg}/m^3$, $a_2 = a_{co} = 3488\ \mathrm{k}m$. The effect of time dilation at the center of the Earth will be the result of the superposition of the fields of the two spheres. Its proper time taking into account equation (34) with constant (6) will be

$$\tau = \left[1 - \frac{4\pi\gamma}{3c^2}\left(a_1^2\rho_1 + a_2^2\rho_2\right)\right] t. \tag{35}$$

The relative difference between the time in the absence of gravity and in the center of the Earth will be $(t_0 - \tau_c)/t_0 = 8{,}05 \cdot 10^{-10}$. The course of time in the center will differ little from the course of time in the entire region of the core. In the region of the mantle, it will begin to approach the surface. The relative difference between the clock readings in the center of the Earth and on the surface is $(t_s - \tau_c)/t_s = 1{,}1 \cdot 10^{-10}$. If we consider the age of the Earth to be $T_3 = 4.54 \cdot 10^9$ yr, then the center of the Earth will be 1/2 yr younger than its surface. This result is about 5 times smaller than the value obtained using Newtonian potentials [17].

The course of time below the Earth's surface

Let's find how time will change in the Earth's crust, whose average density is $\rho_{cr} = 2800\ \mathrm{k}g/\mathrm{m}^3$. We consider the gravitational field created by two spheres, one of which has the radius of the Earth and density ρ_{cr}. Mass of the second sphere is equal to the remainder part of the Earth $M_2 = 2.94 \cdot 10^{\,24}\ kg$ and its radius is the radius of the Earth, reduced by the average thickness of the crust a_{cr}. The time dilation in this system within the crust will be the total result of the gravitational field created by the first sphere with the metric (33) and the Schwarzschild field with metrical functions (9) created by the second sphere

$$\Delta\tau = \left[1 - \frac{\gamma}{c^2}\left(\frac{4\pi}{3}a_1^2\rho_{cr} + \frac{M_2}{r}\right)\right]\Delta t, \tag{36}$$

where the distance to the center of the Earth is in the interval $a_1 \geq r \geq a_1 - a_\kappa$. Within the Earth's crust, with $r = a_1 + l$ it can be rewritten as

$$\Delta\tau = \left[1 - \frac{\gamma}{c^2}\left(\frac{4\pi}{3}a_1^2\rho_{cr} + \frac{M_2}{a_1} + \frac{M_2 l}{a_1^2}\right)\right]\Delta t, \tag{37}$$

where l is the distance to the Earth's surface. Thus, the difference in time intervals measured on the surface of the Earth and in its crust will be

$$\Delta\tau_s - \Delta\tau_{cr} = \frac{\gamma}{c^2}\frac{M_2 l}{a_E^2}\Delta\tau_s \tag{38}$$

due to the approximate equality $\Delta\tau_s \approx \Delta t$.

The course of time below the Earth's surface

Coefficient

$$K_\tau = \frac{\gamma}{c^2}\frac{M_2}{a_E^2} \tag{39}$$

satisfies the relation

$$K_\tau = \frac{\Delta\tau_s - \Delta\tau}{l\Delta\tau_s}\,. \tag{40}$$

Its value determining change of time course depending on depth for the mass M_2 is $K_\tau = 5.36 \cdot 10^{-17}\ m^{-1}$. According to the results obtained in [17] using the linear dependence of time on the gravitational potential, the coefficient (40) is about $K_\tau = 1.1 \cdot 10^{-16}\ m^{-1}$. Most of the Earth's surface is covered by the ocean, and if we substitute the water density $\rho_w = 1020\ \mathrm{k}g/\mathrm{m}^3$ as the density in (36), considering it as the density of the first sphere, then the mass of the second sphere will be $\widetilde{M}_2 = 4.86 \cdot 10^{24}\ kg$. This gives in equation (39) coefficient $\widetilde{K}_\tau = 8.86 \cdot 10^{-17}\ m^{-1}$. This result can be verified using atomic clocks, the precision of which [18] allows one to determine the effect of time dilation beneath the surface of the Earth's crust or the ocean. The result can be significantly affected by the deviation of the density from the average in different regions, in contrast to the model with the dependence of the flow of time only on the gravitational potential.

Summary

- Matter, even static, warps space, and it is natural to assume that the result is a vacuum pressure. We have examined a possible mechanism for the occurrence of this pressure, based on the gravitational defect of the masses and the assumption of elasticity of space in compliance with the law of energy conservation. The spherical sources of gravity with constant densities and identical gravitational masses is considered in the spheres with the same volume in the remote frame. With the increase in mass defect, the difference between the proper volume of the spheres and their volume in the remote frame increases, which gives a positive pressure of gravitational field and the vacuum pressure equal to it in magnitude but of the opposite sign. In statics, according to the theory of elasticity, the vacuum pressure balances impact of gravity on vacuum. The equation of state for a vacuum containing a distributed locally isotropic static gravity source is $w = -1/3$.
- The resulting vacuum pressure corresponds to the solution of Einstein's equations, in which the course of time inside the sphere is constant. Obtained metric is used to determine the course of time below the Earth's surface. The difference in age between the center of the Earth and its crust turns out to be 5 times smaller than that obtained under the assumption that the flow of time depends only on Newtonian potentials.

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Thank you for your attention!