Large-scale virial relations in 5D Absolute Parallelism with 4th-order gravity

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1. Introduction

Special relativity (SR) unites space and time but does not explain any field or particle. General relativity (GR) relates gravity to space-time curvature; the other fields/particles form the energy-momentum tensor, EMT, and remain unexplained. Einstein wasn't content with GR; he compared the GR-equation sides with a marble palace (the LHS, Einstein's tensor $G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu}R/2,\ G_{\mu\nu;\nu} \equiv 0$) and an old shed (the RHS with the EMT). Later Einstein explored the frame field $h^a{}_\mu(x^\nu)$, $g_{\mu\nu} = \eta_{ab}h^a{}_\mu h^b{}_\nu$, and second order equations which symmetry unites symmetries of both SR (Latin indexes; η_{ab} is Minkowski's metric) and GR (Greek ones) – the third (or united) relativity, but Einstein call it Absolute Parallelism (AP).

The list of compatible 2^d-order AP equations (found by A. Einstein and W. Mayer in [0]) includes the two-parameter class of Lagrangian equations and three more classes. And there exists the exceptional equation (EE), non-Lagrangian, which solutions don't allow co-singularities (the principal terms do not remain regular for one-degenerate co-frame matrices), and, if $D{=}5$, contra-singularities (degenerate contra-frame density of some weight) [1]:

$$\mathbb{E}_{a\mu}\!:\ L_{a\mu\nu;\nu} - \tfrac{1}{3}(f_{a\mu} + L_{a\mu\nu}\Phi_\nu) = 0,\ \mathbb{E}_{a\mu;\mu}\!:\ f_{\mu\nu;\nu} = \tfrac{1}{2}S_{\mu\nu\lambda}f_{\lambda\nu}\,[= (S_{\mu\nu\lambda}\Phi_\lambda)_{;\nu}];$$

here $L_{a\mu\nu} = \Lambda_{a\mu\nu} - S_{a\mu\nu} - \tfrac{2}{3}h_{a[\mu}\Phi_{\nu]},\ \ \Lambda^a{}_{\mu\nu} = h^a{}_{\mu;\nu} - h^a{}_{\nu;\mu}$

$(\Lambda^a{}_{[\mu\nu;\lambda]} \equiv 0)$, $S_{\mu\nu\lambda} = 3\Lambda_{[\mu\nu\lambda]}$, $\Phi_\nu = h^\mu_a\Lambda^a{}_{\mu\nu}$, $f_{\mu\nu} = 2\Phi_{[\mu;\nu]}$.

The EE doesn't allow $D{=}4$ (we need at least one extra dimension!):

$$\mathbb{E}_{\mu\mu} = \mathbb{E}^a{}_\mu h_a^{\ \mu} = \tfrac{4-D}{3}\Phi_{\mu;\mu} - \tfrac{1}{2}\Lambda^2_{abc} + \tfrac{1}{3}S^2_{abc} + \tfrac{D-1}{9}\Phi^2_a = 0.$$

2. EE's features

The theory has a number of key features (remember, $D{=}5$):

⊛ 15 polarizations have very different functions and amplitudes (four classes; generally, a higher class means many orders smaller amplitudes) as they relate to different irreducible parts of Λ and Λ', such as Φ_μ (3+1 pol-s, 2^d- and 1^{st}-class), $S_{\mu\nu\lambda}$ (3 pol-s, 1^{st}-class), and the Riemannian curvature tensor (or the Weyl tensor; 5 pol-s, 3^d-class);

⊛ three unstable pol-s (0^{th}-class) grow linearly under action of three stable pol-s relating to $f_{\mu\nu}$ (2^d-class), while h'^2-terms are tiny:

$\Lambda_{\lambda\mu\nu;\tau;\tau} = -\tfrac{2}{3}f_{\mu\nu;\lambda} + (\Lambda\Lambda', \Lambda^3)$ ($\Box S \approx \Box\Phi \approx 0 \approx \Box f$ — stable pol-ns);

⊛ non-stationary O_4-symmetrical solutions exist which resemble a (single) longitudinal wave in Chaplygin gas [2] (the 1^{st}-class pol-n relating to Φ_μ; others don't survive in this symmetry); the wave can serve as a cosmological shallow waveguide for tangential shorter waves, with ultrarelativistic expansion and different evolution of waves amplitudes – according to the structures of quadratic terms (whether they include 0^{th}-class parts or only lower class ones);

⊛ non-linear localised h-field configurations can carry digital information – topological charges and (for symmetrical configurations) quasi-charges (when 0^{th}-class waves become large enough), and a QM-like $4D$-phenomenology emerges through averaging along the huge extra-dimension, along a length L, the width of large-scale O_4-wave in co-moving coordinates [2]; note, two thin lines in a 4d-space have tiny chances to intersect in a single approach. The complete description is five-dimensional, not four!

3. Forth order gravity

The proper EMT (where only f-polarizations play the role) appears with the prolonged, forth-order symmetrical equation $[\mathbb{E}_{(\mu\nu);\tau;\tau}]$:

$$G_{\mu\nu;\tau;\tau} + G_{\epsilon\tau}(2R_{\epsilon\mu\tau\nu} - g_{\mu\nu}R_{\epsilon\tau}/2) = -\tfrac{2}{9}T^{(f)}_{\mu\nu} + B_{[\mu\rho][\nu\tau]}(\Lambda^2)_{;\rho;\tau},$$

where $T^{(f)}_{\mu\nu} = f_{\mu\tau}f_{\nu\tau} - \frac{1}{4}g_{\mu\nu}f^2_{ab}$;

this eq-n follows also from a "Lagrangian" quadratic in the field equation $\mathbb{E}_{(\mu\nu)}$. Note the absence of free parameters. The scale L is a parameter of solution, not the theory; and L reduces to the Planck length λ_P via multiplying by a tiny factor relating to the amplitude of f-waves. In other words, the "*conventional scale* of EMT" (where the "conventional energy" of a photon is just its angular frequency ω, in natural units with $c = 1 = \hbar$) differs very-very much from the authentic scale of $T^{(f)}_{\mu\nu}$.

So, one can consider the static 4d-equation $\Delta^2\varphi(x^\alpha) = -\rho(x^\alpha)$ and suggest that masses are very extended along the extra dimension, the length L. A point "mass" $aR^{-3}\delta(R)$ gives the next solution (every large mass/over-density is accompanied by an under-density, so the logarithmic growth should stop somewhere)

$\varphi(R) = a/8\ln(R^2) - b/R^2$; we discard $+c + d\,R^2$.

For an L-extended mass m, at scales $r \ll L$ one gets the Newton acceleration $\varphi'_N = G_N m/r^2$, while for large distances $r \gg L$ it looks different: $\varphi' = G_N m/(rL)$. The second-order equation also should be accounted for and this restricts the solutions; at some constraint on the set a, b the first correction to Newton's force (the Rindler term) $\Delta\varphi \sim r/L^2$ can vanish, so

$\Delta\varphi(r \ll L) = \varphi - \varphi_N \sim r^2/L^3$ [2].

It seems L should be about (few) hundred parsecs.

4. Large scale virial relations

At large scales, if generally $|\vec{r}_{ij}| \gg L$, a virial relation takes the form

$$(<\psi>_T = \tfrac{1}{T}\int_{t0}^{T+t0}\psi\,dt): \quad \left\langle \sum_i (\dot{\vec{r}}_i\vec{p}_i + \vec{r}_i\dot{\vec{p}}_i) \right\rangle_T \approx 0 \quad \Rightarrow$$

$$<\sum_i m_i\gamma_i v_i^2>_T = \left\langle \sum_{i,j\neq i} \vec{r}_i \frac{\partial U_{ij}(\vec{r}_i - \vec{r}_j)}{\partial \vec{r}_i} \right\rangle_T \approx \frac{G}{L}\left[(\sum_i m_i)^2 - \sum_i m_i^2\right].$$

This relation can be applied both to galaxy clusters and to large gas clouds. The (baryon) mass of a galaxy $M_{\rm gal}$ can hardly be measured accurately, and the function $L_{\rm gal}(M_{\rm gal})$ can be non-linear [3], $L_{\rm gal} \sim M^2_{\rm gal}$ (for stars $L_{\rm st} \sim M^4_{\rm st}$) – this would support the suggested theory (see also DarkMatterCrisis.wordpress.com).

5. References

[0] Einstein A., Mayer W. Systematische Untersuchung über kompatible Feldgleichungen, welche in einem Riemannschen Raume mit Fernparallelismus gesetzt werden können. *Sitzungsber. preuss. Akad. Wiss., phys.-math. Kl.*, 1931, 257–265. (Einstein's Full Collection. V. 2. 353–365. Rus.)

[1] Zhogin I. Proc. PIRT-2011. Moscow: BMSTU, 2012, p. 337; arXiv:gr-qc/1109.1679 / Thesis (book); arXiv:gr-qc/0412130 / One more SNeIa fitting (D=5); arXiv:gr-qc/0902.4513.

[2] Zhogin I. AP: Spherical symmetry; arXiv: gr-qc/0412081v3 / Topological charges and quasi-charges; arXiv:gr-qc/0610076 / Large-scale change in Newton's law; arXiv: gr-qc/0704.0857.

[3] Valageas P. and Schaeffer R. The mass and luminosity functions of galaxies and their evolution. *Astron. Astrophys.*, 1999, vol. 345, p. 329; arXiv: astro-ph/9812213.

A1. One simple AP equation and its compatibility

A simple (perhaps the simplest) compatible AP equation looks simply

$\mathbb{E}^*_{a\mu} = \Lambda_{a\mu\nu;\nu} = 0;$ it's compatibility is ensured by the identity

$\mathbb{E}^*_{a\mu;\mu} = \Lambda_{a\mu\nu;\nu;\mu} \equiv 0$; or $(h\mathbb{E}^{*\mu}_a)_{,\mu} = (h\Lambda_a{}^{\mu\nu})_{,\nu\mu} \equiv 0$ $(h = \det h^a{}_\mu)$.

The linearized simple equation $\mathbb{E}^*_{a\mu}$ (SE; together with the linearized Λ-identity $\Lambda_{a[\mu\nu;\lambda]} \equiv 0$ – if one would like to use the first order system for Λ's) looks like a D-fold Maxwell system (the every value of scalar index a gives its own Maxwell-like system): $\mathbb{E}^{*a}{}_\mu \approx h^a{}_{\mu,\nu\nu} - h^a{}_{\nu,\mu\nu} = 0.$

It should follow from the compatibility theory (as it's formulated by J. F. Pommaret in [a1]) that the symbol G_2 of this system (or symbol G_1 for the first-order linearized system with Λ's) is involutive, and all further identities should be valid automatically (and the system is compatible).

The other "divergence" $\mathbb{E}^{*a}{}_{\mu;\nu} h_a{}^\nu = 0 = \mathbb{E}^{*a}{}_{\mu,a}$ gives the next (Maxwell-like) equation (one should use the contracted Λ-identity $\Lambda_{abc,a} + f_{bc} \equiv 0$):

$\mathbb{E}^*_{a\mu,a} = f_{\mu\nu;\nu} - J_\mu(\Lambda\Lambda') = 0;$

so (for compatibility) this current should be conserved, i.e. the next identity must be (automatically!) valid: $J_{\mu;\mu} \equiv 0$. How to explain this?

One can separate symmetric and skew-symmetric parts of any AP eq-n (2^d-order; for SE σ=0, τ=1) as follows (we use the Einstein tensor; $G_{\mu\mu;\nu} \equiv 0$):

$$\mathbb{E}_{[\mu\nu]}: \; S_{\mu\nu\lambda;\lambda} + \tau f_{\mu\nu} + V_{[\mu\nu]}(\Lambda^2) = 0\,; \; \mathbb{E}_{(\mu\nu)}: \; \Lambda_{(\mu\nu)\lambda;\lambda} + \sigma(\Phi_{(\mu;\nu)} - g_{\mu\nu}\Phi_{\lambda;\lambda}) + (\Lambda^2) =$$
$$= -G_{\mu\nu} + (\sigma - 1)(\Phi_{(\mu;\nu)} - g_{\mu\nu}\Phi_{\lambda;\lambda}) + V_{(\mu\nu)}(\Lambda^2) = 0\,;$$

these parts give two Maxwell-like equations which currents should be identical (the first identity necessary for compatibility; $\sigma \neq 1, \tau \neq 0$):

$$(\sigma - 1)[f_{\mu\nu;\nu} - J^{(s)}_\mu(\Lambda\Lambda')] = 0, \; \tau[f_{\mu\nu;\nu} - J^{(a)}_\mu(\Lambda\Lambda')] = 0, \; J^{(s)}_\mu \equiv J^{(a)}_\mu \propto V_{[\mu\nu];\nu}.$$

Evidently, if $\tau \neq 0$, the current is trivial as following from the skew-symmetric part. The exception is $\tau = 0$, with the exceptional equation (EE) where the current (it's trivial too) follows only from the symmetric part. Coulomb-like $f_{\mu\nu}$-field needs a long extra-dimension (Einstein had mentioned troubles with an f-candidate in AP to somebody, maybe to A. Pais [a2]).

For notes

We use a sort of "equivalence class notation":

$A_{\mu\cdots;\nu\cdots\mu} := \{A^\mu{}_{\cdots;\nu\cdots\mu}\,,\; A_{\mu\cdots;\nu\cdots}{}^\mu\}$

– no matter which of two contracted indexes to move up.

[a1] J. F. Pommaret, *Systems of Partial Differential Equations and Lie Pseudogroups,* Math. and its Applications, Vol. 14 (New York, 1978).

[a2] A. Pais, Subtle is the Lord,