EIGENFREQUENCIES OF ROTATING WAVEGUIDES AND RESONATORS

Daria Titova
Southern Federal University
daria.titova1@gmail.com
Open for discussions!

Abstract & Motivation

Problems of electrodynamics in rotating reference frames are of great importance for studying the postulates of the General Relativity. The results of solving such problems are necessary to explain experiments on electromagnetic fields in rotating interferometers and gyroscopes [1], [2], [3]. Known solutions to such problems were obtained on the basis of various kinds of assumptions that do not allow taking into account the effect of the equivalent gravitational field on the electromagnetic field in rotating (non-inertial) reference frames. This led to conclusions, that are equivalent to those in classical electrodynamics [4], [5], [6], [7], [8], [9], [10], [11], [12], [13].

Rigorous electrodynamic theory of rotating interferometers and gyroscopes based on the electrodynamics of General Relativity was described in [14], where it was shown that the parameters of the electromagnetic field in inertial and non-inertial reference frames are different. For internal electromagnetic problems, it was shown that due to rotation, cutoff frequencies of the waveguide at rest ω_0 are split into two new cutoff frequencies $\omega_n = \omega_0 \pm n\Omega$, where n is the order of the mode propagating in the waveguide (which allowed for obtaining a rigorous electrodynamic explanation of the Sagnac effect [15]). And on the basis of a rigorous solution of the problem of the existence of an electromagnetic field in a rotating cavity resonator, an expression was obtained for the splitting of the eigenfrequency $\omega_n = \omega_0 \pm n\Omega$, (which was also shown by approximate methods in [6], [7] and coincides with the experimental results). However, in reality, in the presence of an oscillation source, it is necessary to formulate and solve a problem of excitation

In this paper, a rigorous formulation and solution of the problem of excitation of an electromagnetic field in a rotating waveguide and resonator (which are mathematical models of interferometers and gyroscopes) is solved using covariant form of Maxwell equations independent of the space metric [14]. The electromagnetic field strength vectors are represented as tensors and tensor densities of weight +1. The statement and solution of the problem is carried out in a rotating frame of reference in which the cavities, the dielectric filling, and the source located in them are at rest. The non-inertial frame of reference itself rotates relative to inertial reference frame. Electromagnetic field in the waveguide and resonator is represented as the sum of the primary electromagnetic field excited by the oscillation sources and secondary electromagnetic field reflected from PEC walls of the cavities. Components of the electromagnetic field vectors of TM and TE types are represented via Debye potentials that satisfy wave equation in a rotating reference frame. It is shown that during rotation, new terms appear in the components of electromagnetic field vectors, which are proportional to the rotation rate of the cavity, and the wave number depends on the rotation rate. The latter leads to the effects of splitting of resonator eigenfrequencies and waveguide cutoff frequencies due to rotation under the influence of the Coriolis force and centrifugal force.

Electromagnetic Equations in Rotating Reference Frames [14]

General principles for constructing a solution to the problems of excitation of electromagnetic field in rotating cavities

Inertial reference frame — Rotating (non-inertial) reference frame

$$K'(x',y',z',iv_\phi t) = K'(r',\varphi',z',iv_\phi t)^* \qquad K(x,y,z,t) = K(r,\varphi,z,t)^*$$

$$r' = r,\ \varphi' = \varphi + \Omega t,\ z' = z$$

$$v_\phi = 1/\sqrt{\varepsilon\mu}$$

Maxwell equations:

$$rot\hat{\bar{H}} = \frac{\partial \hat{\bar{D}}}{\partial t} + \hat{\bar{j}}^E, \quad rot\bar{E} = \frac{-\partial \bar{B}}{\partial t} - \bar{j}^H, \quad div\hat{\bar{D}} = \hat{\rho}^E, \quad div\bar{B} = \rho^H,$$

$\bar{E} = E_\alpha = (E_1, E_2, E_3) = (E_r, rE_\varphi, E_z)$ is covariant vector;
$\hat{\bar{H}} = \hat{H}^{\alpha\beta} = (\hat{H}^{23}, -\hat{H}^{13}, \hat{H}^{12}) = (H_r, rH_\varphi, H_z)$ is contravariant bivector density of weight +1;
$\hat{\bar{D}} = \hat{D}^\alpha = (\hat{D}^1, \hat{D}^2, \hat{D}^3) = (r\hat{D}^r, \hat{D}^\varphi, r\hat{D}^z)$ is contravariant vector density of weight +1;
$\bar{B} = B_{\alpha\beta} = (B_{23}, -B_{13}, B_{12}) = (rB_{r\varphi}, B_{Rz}, rB_{\varphi z})$ is covariant bivector;
$\hat{\bar{j}}^E = \hat{j}^{E,\alpha} = (r\hat{j}^{E,r}, \hat{j}^{E,\varphi}, r\hat{j}^{E,z})$ – contravariant vector density; $\hat{\rho}^E$ is scalar density;
$\bar{j}^H = j^H_{\alpha\beta} = (r\hat{j}^{H,r}, \hat{j}^{H,\varphi}, r\hat{j}^{H,z})$ is bivector density; ρ^H is simple scalar density
[4], [5], [21]

Constitutive equations:

$$\hat{\bar{D}} = \varepsilon(\eta^2\bar{E}_\perp + \bar{E}_\parallel + W\eta^2[\hat{\bar{H}}\beta]), \quad \bar{B} = \mu(\eta^2\hat{\bar{H}}_\perp + \hat{\bar{H}}_\parallel + W^{-1}\eta^2[\beta\bar{E}]),$$

$\bar{E}, \bar{B}$ - tensors, $\hat{\bar{D}}, \hat{\bar{H}}$ - tensor densities of weight +1 ;
$\eta = (1-\beta^2)^{-1/2}$, symbols denote $\parallel$ and $\perp$ components of electromagnetic field vectors, that coincide and do not coincide on index with the components $\bar{v} = [\bar{\Omega}\bar{r}]$, $\beta = vc^{-1}$,
$W = \sqrt{\mu/\varepsilon}$ is characteristic impedance; $v_\phi = 1/\sqrt{\varepsilon\mu}$ is phase velocity in the medium

Wave equation in rotating reference frames:

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial V^{E,H}}{\partial r} + \frac{1-\beta^2}{r^2}\frac{\partial^2 V^{E,H}}{\partial\varphi^2} + \frac{2\beta}{v_\phi r}\frac{\partial^2 V^{E,H}}{\partial\varphi\partial t} + \frac{\partial^2 V^{E,H}}{\partial z^2} - \frac{1}{v_\phi^2}\frac{\partial^2 V^{E,H}}{\partial t^2} = 0$$

Debye potentials in rotating reference frames:

$$V^E = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2}U_n^E dæ, \qquad V^H = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2}U_n^H dæ$$

Wavenumber in rotating reference frame: $k_n = \frac{\omega_n}{v_\phi} = \frac{\omega_0 + n\Omega}{v_\phi} = k_0 + \frac{n\Omega}{v_\phi}$

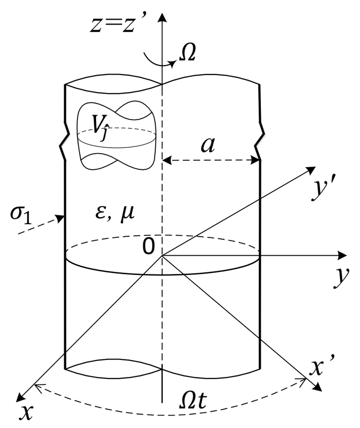
*Ratios for the world time t and coordinate time τ in reference frames K' and K are expressed in non-inertial reference frame as $\tau = t\sqrt{-g_{00}}/v_\phi$, where g_{00} is the component of the metric tensor g_{ik} for time. g_{ik} relates coordinates in non-inertial reference frames as $ds^2 = g_{ik}dx^i dx^k$, where ds^2 is the squared distance in metric space (the interval between events), $dx^i dx^k$ are the coordinate derivatives. g_{00} is represented $g_{00} = -c^2(1-(\Omega R)^2/c^2) = -c^2(1-\beta^2)$, where $\beta = \Omega R/v_\phi$ in non-inertial reference frame K, and $g_{00} = 1$ in inertial reference frame K'.

Rotating Waveguides

Abbott B.P., Abbott R., Abbott T.D., Abernathy M.R. et al. (LIGO Scientific Collaboration and Virgo Collaboration), «Observation of Gravitational Waves from a Binary Black Hole Merger,» Physical Review Letters, т. 6, № 116, 2016.

E. Toldi , H. Lefèvre, F. Guattari, A. Bigueur, A. Steib, D. Ponceau, C. Moluçon, E. Ducloux, J. Wassermann, U. Schreiber, "First steps for a Giant FOG: Searching for the limits," Inertial Sensors and Systems 2017, Karlsruhe, Germany.

Mathematical Model of Interferometer - Rotating Waveguide:



Waveguide Parameters:
radius $r = a$,
walls conductivity σ_1,
$\varepsilon = \varepsilon_0\varepsilon_r = \varepsilon' - i\varepsilon'' = \varepsilon_0(\varepsilon_r' - i\varepsilon_r'')$
$\mu = \mu_0\mu_r$, $\mu_r = 1$
measured in rotating reference frame K.

Total Exited EM field in rotating Waveguide:

$$E^E = E^{E,i} + E^{E,S}, \qquad H^E = H^{E,i} + H^{E,S},$$
$$E^H = E^{H,i} + E^{H,S}, \qquad H^H = H^{H,P} + H^{H,S}.$$

$E^{E,H,i}$, $H^{E,H,i}$ is the primary (incident) EM field, exited by the arbitrary sources and charges in the domain $V_{\hat{j}}$,
$E^{E,H,S}$, $H^{E,H,S}$ id the secondary (scattered) EM field excited by equivalent surface currents and charges at the waveguide walls

Cylindrical Expansion for Rotating Waveguide Debye Potentials:

$$U_n^{E,H} = U_n^{E,H,i} + U_n^{E,H,S} = e^{-in\varphi}e^{\pm\sqrt{æ^2-k_n^2}z}\frac{æ}{\sqrt{æ^2-k_n^2}}\begin{Bmatrix} H_n^{(2)}(ær)F_n^{(2)E,H}(æ) + a_n^{E,H}J_n(ær) \\ J_n(ær)F_n^{(1)E,H}(æ) \end{Bmatrix}, \quad \begin{matrix} r > \rho, \\ r < \rho, \end{matrix} \quad (1)$$

Boundary Value Problem at $r = a$: $\sigma_1 = \infty$

$$a_n^E = -\frac{H_n^{(2)}(æa)}{J_n(æa)}F_n^{(2)E}, \qquad a_n^H = -\frac{\left(H_n^{(2)}(æa)\right)'}{\left(J_n(æa)\right)'}F_n^{(2)H}, \quad r = a.$$

æ – transverse propagation coefficient of the azimuthal harmonic of the EM field
n are the order of the Bessel function and the order of the excited mode,
$F_n^{(s),E,H}$ - coefficients determined by arbitrary currents and charges of the oscillation sources [14]
$\chi_n = \sqrt{k_n^2 - æ^2}$ is the longitudinal propagation coefficient

Electromagnetic Field in Rotating Cylindrical Waveguide

***TM*-field:**

$$\beta = \Omega r/v_\phi$$

$$E_r^E = \sum_{n=-\infty}^{\infty}E_{rn}^E = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2}\frac{\partial^2 U_n^E(æ)}{\partial r\partial z}dæ,$$
$$E_\varphi^E = \sum_{n=-\infty}^{\infty}E_{\varphi n}^E = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2 r}\frac{\partial^2 U_n^E(æ)}{\partial\varphi\partial z}dæ,$$
$$E_z^E = \sum_{n=-\infty}^{\infty}E_{zn}^E = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{æ^2 - K_n k_n}{æ^2}U_n^E(æ)dæ, \quad (2)$$
$$H_r^E = \sum_{n=-\infty}^{\infty}H_{rn}^E = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{i\omega_0\varepsilon}{æ^2 r}\frac{\partial U_n^E(æ)}{\partial\varphi} + W^{-1}\frac{1}{æ^2}\frac{\partial}{\partial r}\beta\frac{\partial U_n^E(æ)}{\partial r}\right)dæ,$$
$$H_\varphi^E = \sum_{n=-\infty}^{\infty}H_{\varphi n}^E = -e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{i\omega_0\varepsilon}{æ^2}\frac{\partial U_n^E(æ)}{\partial r} - \frac{W^{-1}\beta}{æ^2 r}\frac{\partial^2 U_n^E(æ)}{\partial\varphi\partial r}\right)dæ,$$
$$H_z^E = \sum_{n=-\infty}^{\infty}H_{zn}^E = W^{-1}\beta E_r^E;$$

***TE*-field:**

$$E_r^H = \sum_{n=-\infty}^{\infty}E_{rn}^H = -e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{i\omega_0\mu}{æ^2 r}\frac{\partial U_n^H(æ)}{\partial\varphi} + W\frac{1}{æ^2}\frac{\partial}{\partial r}\beta\frac{\partial U_n^H(æ)}{\partial r}\right)dæ,$$
$$E_\varphi^H = \sum_{n=-\infty}^{\infty}E_{\varphi n}^H = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{i\omega_0\mu}{æ^2}\frac{\partial U_n^H(æ)}{\partial r} - \frac{W\beta}{æ^2 r}\frac{\partial^2 U_n^H(æ)}{\partial\varphi\partial r}\right)dæ,$$
$$E_z^H = \sum_{n=-\infty}^{\infty}E_{zn}^H = -\beta W H_r^H, \quad (3)$$
$$H_r^H = \sum_{n=-\infty}^{\infty}H_{rn}^H = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2}\frac{\partial^2 U_n^H(æ)}{\partial r\partial z}dæ,$$
$$H_\varphi^H = \sum_{n=-\infty}^{\infty}H_{\varphi n}^H = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2 r}\frac{\partial^2 U_n^H(æ)}{\partial\varphi\partial z}dæ,$$
$$H_z^H = \sum_{n=-\infty}^{\infty}H_{zn}^H = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{æ^2 - K_n k_n}{æ^2}U_n^H(æ)dæ.$$

Clockwise and counterclockwise rotating partial harmonics E_{np} of EM field:

$$E_z^E = \sum_{n=-\infty}^{\infty}e^{-in\varphi}E_{zn1}^E + \sum_{n=-\infty}^{\infty}e^{-in\varphi}E_{zn2}^E + \dots\sum_{n=-\infty}^{\infty}e^{-in\varphi}E_{znp}^E =$$
$$= E_{z01}^E + E_{z02}^E + E_{z03}^E + \dots + e^{-in\varphi}\left(E_{z11}^E + E_{z12}^E + E_{z13}^E + \dots\right) + e^{in\varphi}\left(E_{z-11}^E + E_{z-12}^E + E_{z-13}^E + \dots\right) + \dots$$

n are the order of the Bessel function and the order of the excited mode,
$p = 1,2,3 \dots$ order of the root $u_{np}^E = æa$ of the dispersion equation $J_n(æa) = 0$ for TM field

Longitudinal wave propagation coefficient: $\chi_n = \sqrt{\left(\frac{\omega_0 + n\Omega}{v_\phi}\right)^2 - æ^2}$

Cutoff wavelength: $\lambda_{0,c}^+ = \frac{2\pi a}{u_{np}^E - a|n|\Omega/v_\phi}$, $\qquad \lambda_{0,c}^- = \frac{2\pi a}{u_{np}^E + a|n|\Omega/v_\phi}$,

$u_{np}^E = æa$ are roots of order $p = 1,2,3 \dots$ of the dispersion equation $J_n(æa) = 0$ for TM field (1),
u_{np}^H are roots of order $p = 1,2,3 \dots$ of the dispersion equation $(J_n(æa))' = 0$ for TE field (1)

Cutoff frequency at rest [14]:

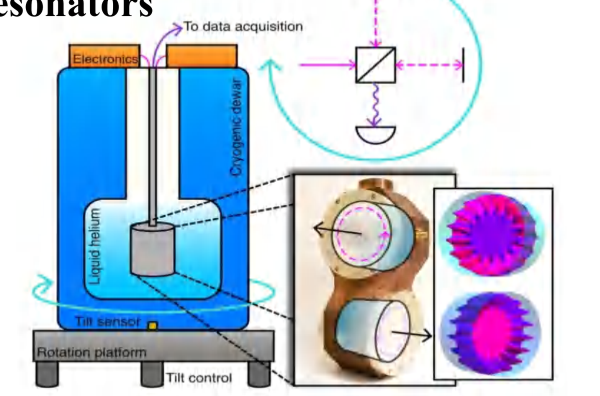
$$\omega_{0,np} = \frac{u_{np}^E v_\phi}{a}.$$

Cutoff frequency under rotation [14]:

$$\omega_{0,np}^{+c} = \frac{u_{np}^E v_\phi}{a} - n\Omega, \qquad \omega_{0,np}^{-c} = \frac{u_{np}^E v_\phi}{a} + n\Omega.$$

Rotating Resonators

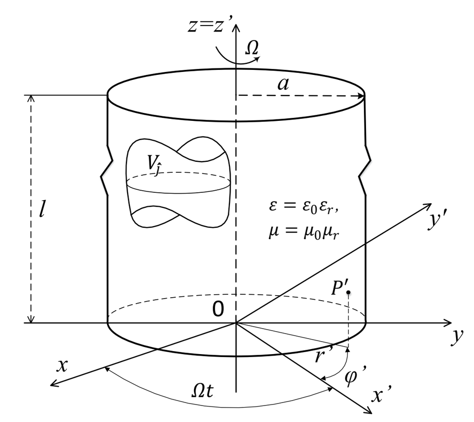
S. Herrmann, A. Senger, K. Möhle, M. Nagel, E. V. Kovalchuk, and A. Peters, "Rotating optical cavity experiment testing Lorentz invariance at the 10 (-17) level" Physical Review D, vol. 80, N 10, 2009.



M. Nagel, S. Parker, E. Kovalchuk et al., "Direct terrestrial test of Lorentz symmetry in electrodynamics to 10^(−18)," Nature Communications , vol. 6, 2015

Mathematical Model of Gyroscope - Rotating Cavity Resonator:

Cavity resonator parameters:
Radius a, length $l = 5a$
Walls conductivity σ_1,
$\varepsilon = \varepsilon_0\varepsilon_r = \varepsilon' - i\varepsilon'' = \varepsilon_0(\varepsilon_r' - i\varepsilon_r'')$
$\mu = \mu_0\mu_r$, $\mu_r = 1$
Measured in reference frame K.



Total Exited EM field in rotating Resonator:

$$E^E = E^{E,i} + E^{E,S}, \qquad H^E = H^{E,i} + H^{E,S},$$
$$E^H = E^{H,i} + E^{H,S}, \qquad H^H = H^{H,P} + H^{H,S}.$$

$E^{E,H,i}$, $H^{E,H,i}$ is the primary (incident) EM field exited by the arbitrary sources and charges in the domain $V_{\hat{j}}$,
$E^{E,H,S}$, $H^{E,H,S}$ id the secondary (scattered) EM field excited by equivalent surface currents and charges at the cavity walls

Cylindrical Expansion for Rotating Resonator Debye Potentials:

$$U_n^{E,H} = U_n^{E,H,i} + U_n^{E,H,S} = e^{-in\varphi}\frac{æ}{\sqrt{æ^2-k_n^2}}\begin{Bmatrix} H_n^{(2)}(ær)F_n^{(2)E,H}(æ)Z_n^E(z) + a_n^{E,H}J_n(ær)Z_n^E(z) \\ J_n(ær)F_n^{(1)E,H}(æ)Z_n^E(z) \end{Bmatrix}, \quad \begin{matrix} r > \rho \\ r < \rho, \end{matrix}$$

Boundary Value Problem at $z = 0, l$: $\sigma_1 = \infty$

$$Z_n^E(z) = g_{1n}2\cos\left(\frac{q\pi}{l}z\right), \qquad Z_n^H(z) = -i2g_{1n}\sin\left(\frac{q\pi}{l}z\right),$$

$\frac{q\pi}{l}$ are roots of $\sin\left(\sqrt{k_n^2 - æ^2}l\right) = 0$, $q = 0, 1, 2 \dots$.
g_{1n} are constant factors
$H_n^{(2)}(x), J_n(x)$, are Hankel and Bessel functions, respectively

Electromagnetic Field in Rotating Cylindrical Cavity Resonator

***TM*-field:**

$$\beta = \Omega r/v_\phi$$

$$E_r^E = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}E_{r,npq}^E = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2}\frac{\partial^2 U_{npq}^E(æ)}{\partial r\partial z}dæ,$$
$$E_\varphi^E = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}E_{\varphi,npq}^E = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2 r}\frac{\partial^2 U_{npq}^E(æ)}{\partial\varphi\partial z}dæ,$$
$$E_z^E = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}E_{z,npq}^E = \frac{1-\beta^2}{\varepsilon}\hat{D}^z - \beta W H_r^E, \quad (4)$$
$$H_r^E = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}H_{r,npq}^E = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}\int_{-\infty}^{\infty}\left(\frac{i\omega_0\varepsilon}{æ^2 r}\frac{\partial U_{npq}^E(æ)}{\partial\varphi} + W^{-1}\frac{1}{æ^2}\frac{\partial}{\partial r}\beta\frac{\partial U_{npq}^E(æ)}{\partial r}\right)dæ,$$
$$H_\varphi^E = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}H_{\varphi,npq}^E = -e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}\int_{-\infty}^{\infty}\left(\frac{i\omega_0\varepsilon}{æ^2}\frac{\partial U_{npq}^E(æ)}{\partial r} - \frac{W^{-1}\beta}{æ^2 r}\frac{\partial^2 U_{npq}^E(æ)}{\partial\varphi\partial r}\right)dæ,$$
$$H_z^E = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}H_{z,npq}^E = W^{-1}\beta\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}E_{r,npq}^E$$

***TE*-field:**

$$E_r^H = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}E_{r,npq}^H = -e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}\int_{-\infty}^{\infty}\left(\frac{i\omega_0\mu}{æ^2 r}\frac{\partial U_{npq}^H(æ)}{\partial\varphi} + W\frac{1}{æ^2}\frac{\partial}{\partial r}\beta\frac{\partial U_{npq}^H(æ)}{\partial r}\right)dæ,$$
$$E_\varphi^H = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}E_{\varphi,npq}^H = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}\int_{-\infty}^{\infty}\left(\frac{i\omega_0\mu}{æ^2}\frac{\partial U_{npq}^H(æ)}{\partial r} - \frac{W\beta}{æ^2 r}\frac{\partial^2 U_{npq}^H(æ)}{\partial\varphi\partial r}\right)dæ,$$
$$E_z^H = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}E_{z,npq}^H = -\beta W\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}H_{r,npq}^H, \quad (5)$$
$$H_r^H = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}H_{r,npq}^H = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2}\frac{\partial^2 U_{npq}^H(æ)}{\partial r\partial z}dæ,$$
$$H_\varphi^H = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}H_{\varphi,npq}^H = e^{i\omega_0 t}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}\int_{-\infty}^{\infty}\frac{1}{æ^2 r}\frac{\partial^2 U_n^H(æ)}{\partial\varphi\partial z}dæ,$$
$$H_z^H = \sum_{n=-\infty}^{\infty}\sum_{p=1}^{\infty}\sum_{q=0}^{\infty}H_{z,npq}^H = \frac{1-\beta^2}{\mu}B_{r\varphi} + W^{-1}\beta E_r^H$$

Frequency Splitting Effect in Rotating Cavity resonator:

Wavenumber in rotating reference frame: $k_n = \frac{\omega_n}{v_\phi} = \frac{\omega_0 + n\Omega}{v_\phi} = k_0 + \frac{n\Omega}{v_\phi}$

Eigenfrequencies at rest [14]:

$$\omega_0 = v_\phi\sqrt{æ^2 + \left(\frac{q\pi}{l}\right)^2}$$

Eigenfrequencies under rotation [14]:

$$\omega_0^+ = v_\phi\sqrt{æ^2 + \left(\frac{q\pi}{l}\right)^2} - n\Omega, \quad \omega_0^- = v_\phi\sqrt{æ^2 + \left(\frac{q\pi}{l}\right)^2} + n\Omega.$$

Rotation Rate Measurement [20]

$$\Delta\omega = \omega_0^- - \omega_0^+ = 2n\Omega \quad (6)$$

The result (6) coincides with analytical representation for special cases [6], [7] and experimental results [2], [16], [17].

Analysis & Discussion

Expressions (2), (3) show that the EM field consists of an infinite spectrum of inhomogeneous cylindrical waves propagating from the plane z'=ζ. According to (1), at $æ^2 < k_n^2$ the waves are propagating, and at $æ^2 > k_n^2$ they are exponentially decaying. In this case, all components of the field depend on the rotation frequency Ω through the coefficient $\beta = \Omega R/v_\phi$ and the wave number $k_n = \frac{\omega_n}{v_\phi} = \frac{\omega_0 + n\Omega}{v_\phi}$.

Consider an EM field of E-type (2). For $\Omega = 0$, i.e. when the waveguide is stationary, expressions (2) turn into known expressions. At $\Omega \neq 0, \omega_0 \neq 0$ the components H_r^E and H_φ^E change significantly due to the appearance of terms determined by β. The appearance of the longitudinal component H_z^E, proportional to β is due only to the rotation of the waveguide (2). At $\omega_0 = 0$, $\Omega \neq 0$ the wave number $k_n = \frac{\omega_n}{v_\phi} = \frac{n\Omega}{v_\phi}$, that is, all three components of the vector E and vector H remain, but the latter are proportional to the rotation frequency through the coefficient β.

As can be seen from (4), the rotation of a cylindrical resonator leads to the dependence of each spatial harmonic of the EM field excited in it on the rotation frequency Ω through the coefficient $\beta = \Omega r/v_\phi$ and the wave number arguments $k_n = \frac{\omega_n}{v_\phi} = \frac{\omega_0 + n\Omega}{v_\phi}$.

Consider an EM field of E-type (4). For $\Omega = 0$, i.e. when the waveguide is stationary, expressions (2) turn into known expressions. At $\Omega \neq 0, \omega_0 \neq 0$ the components H_r^E and H_φ^E change significantly due to the appearance of terms determined by β. The appearance of the longitudinal component H_z^E, proportional to β is due only to the rotation of the waveguide (4). At $\omega_0 = 0$, $\Omega \neq 0$ волновое число $k_n = \frac{n\Omega}{v_\phi}$, that is, all three components of the vectors E and H, remain, but the latter are proportional to the rotation frequency through the coefficient β. Analysis of the solution showed that in rotating resonators, static electric field ($\omega_0 = 0$) can produce a static ($\omega_0 = 0$) magnetic field with field vectors proportional to rotation rate.

Rigorous solution of the problem of excitation of electromagnetic field in rotating cylindrical cavity resonator and rotating cylindrical waveguide using covariant Maxwell equations allows for further numerical investigation of excited electromagnetic field parameters and cavities spectrum, influenced by relativistic effects.

The obtained solution can be used for setting up, performing and interpreting the results of the experiments with rotating cavity resonators and waveguides [2], [18], [19] and considering new methods for rotation rate measurement [20].

REFERENCES

[1] H. Igel, K.U. Schreiber, A. Gebauer, F. Bernauer, S. Egdorf, A. Simonelli, C.J. Lin, J. Wassermann, S. Donner, C. Hadziioannou, S. Yuan, A. Brotzer, J. Kodet, T. Tanimoto, U. Hugentobler, J.P.R. Wells, "ROMY: a multicomponent ring laser for geodesy and geophysics," Geophysical Journal International, vol. 225, no. 1, pp. 684-698, 2021.
[2] M. Nagel, S. Parker, E. Kovalchuk et al. , "Direct terrestrial test of Lorentz symmetry in electrodynamics to 10^(−18)," Nature Communications , vol. 6, 2015.
[3] K.U. Schreiber and J.P.R. Wells, "Invited Review Article: Large ring lasers for rotation sensing," Review of Scientific Instruments, vol. 84, 2013.
[4] A. Yildiz, C. H. Tang, "Electromagnetic cavity resonances in accelerated systems," Physical Review, vol. 146, no. 4, pp. 947-954, 1966.
[5] E.J. Post, "Sagnac effect," Reviews of Modern Physics, vol. 39, no. 2, pp. 475-493, 1967.
[6] T. Shiozawa, "Phenomenological and electron-theoretical study of the electrodynamics of Rotating Systems," Proceedings of the IEEE, vol. 61, no. 12, p. 1694–1702, 1973.
[7] C.V. Heer, "Resonant frequencies of an electromagnetic cavity in an accelerated system of reference," Physical Review, vol. 134, 1964.
[8] J.V. Bladel , "Electromagnetic fields in the presence of rotating bodies," Proceedings of the IEEE, vol. 64, no. 3, pp. 301-318, 1976.
[9] R. Anderson, H.R. Bilger, G.E. Stedman, "“Sagnac” effect: A century of Earth-rotated interferometers," American Journal of Physics, vol. 62, pp. 975-985, 1994.
[10] H.A. Atwater and T. Shiozawa, "The electromagnetic field in rotating coordinate frames," Proceedings of the IEEE, vol. 63, no. 2, pp. 316-318, 1975.
[11] A. Yildiz, "Electromagnetic cavity resonances in accelerated systems — II," Il Nuovo Cimento B (1965-1970) volume, vol. 61, pp. 1-11, 1969.
[12] E.J. Post, A. Yildiz, "Cavity Resonances in Accelerated Systems," Physical Review Letters, vol. 15, no. 5, pp. 177-178, 1965.
[13] J.L. Anderson, J.W. Ryon, "Electromagnetic radiation in Accelerated Systems," Physical Review, vol. 181, no. 5, pp. 1765-1775, 1969.
[14] B.M. Petrov, Electromagnetic Fields in Rotating Interferometers and Gyroscopes, Moscow: Goryachaya Linia – Telecom, 2015.
[15] B.M. Petrov, "Electrodynamic theory of Sagnac effect," Radioelectronics and Communications Systems, vol. 53, no. 10, pp. 511-520, 2010.
[16] T. Harayama, S. Sunada, and T. Miyasaka, "Wave chaos in rotating optical cavities," Physical Review E, vol. 76, no. 1, 2007.
[17] C.L. Anyi, R.J. Thirkettle, D. Zou, D. Follman, G.D. Cole, K. Ulrich Schreiber, J.-P.R. Wells, "The Macek and Davis experiment revisited: a large ring laser interferometer operating on the 2s2 → 2p4 transition of neon," Applied Optics, vol. 58, no. 2, pp. 302-307, 2019.
[18] Ch. Eisele, A. Yu. Nevsky, and S. Schiller, "Laboratory Test of the Isotropy of Light Propagation at the 10^(-17) level," Physical Review Letters, vol. 103, no. 9, 2009.
[19] P.L. Stanwix, M.E. Tobar, P. Wolf, M. Susli, C.R. Locke, E.N. Ivanov, J. Winterflood, and F. van Kann, "Test of Lorentz Invariance in Electrodynamics Using Rotating Cryogenic Sapphire Microwave Oscillators," Physical Review Letters , vol. 95, no. 4, 2005.
[20] B.M. Petrov, "Resonant method for an object rotation rate measurement and a device that implements the method". Russian Federation Patent RU2562149 C2, 2015.
[21] J.A. Schouten, Tensor analysis for physicists, New York: Dover Publications, Inc., 1989.